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ORIGINAL ARTICLE

Logistics planning and inventory optimization using swarm intelligence: a third party perspective

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Abstract The economic and competitive pressures have made it imperative for organizations to focus on third party or outsourcing to reduce costs and improve operating efficiencies. The thrust of global economy drives the organizations to outsource process, parts, and labor, virtually anywhere in the world and get the desired combination of low cost and high quality. However, ineffective utilization of shipment practices in supply chain prevents to achieve the anticipated outsourcing benefits. In this research, a three-stage inventory model is developed to address the outsourcing issues with different shipment policies between manufacturer, exporter, and assembly point for any manufacturing industry. The model provides a measure for establishing an optimal balance between two conflicting objectives viz. net costs involved and the transportation discounts with consideration of holding cost at all the points in the supply chain. Owing to inherent computational complexities of the problem with higher dimensions, various deterministic approaches practically fail. Proposed work, therefore, utilizes a nature-inspired evolutionary algorithm, namely particle swarm optimization, to solve the problem. This paper applies enhanced particle swarm optimization, a variant of particle swarm optimization for solution purpose. Furthermore, the practical benefits and implications of the proposed model are demonstrated. The results obtained delineate efficacy to handle the fluctuations in the possible shipment options and simultaneously deciding the optimal shipment policies.

Keywords Inventory issue · Multilevel inventory model · Outsourcing · Particle swarm optimization

1 Introduction

Outsourcing represents a major trend not only in manufacturing segments but also in other sectors of corporate world. The key factors that engender the outsourcing of manufacturing jobs are identified to be the low wage rates in countries like China, India, Mexico, etc.; the improved quality with low production costs; relaxation of various international trade barriers; and the reduction in export costs. In order to reduced gap in terms of cost of production and process planning with the competitor, companies are now focusing on effective utilization of logistic practices to increase their margin of profit. Earlier researchers proposed several logistics and shipment policies for different stages of integrated inventory systems. However, integrated inventory system that considers all the factors affecting the inventory management of the outsourcing firms was not addressed. For a single-vendor single-buyer integrated system, Hill [1] presented a shipment policy for determining the vendor's production batch and successive shipments sizes. The model derived the size of the shipment to be a function of the first shipment made, the current shipment number, and the ratio between the production rate and the demand rate. Zhou and Wang [3] developed a production–inventory model for a single-vendor–single-buyer integrated system. It relaxed the assumption made by Hill [1, 2] that the buyer's per unit inventory holding cost per unit time is always greater than that of the vendor. On the other hand, inventory models

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developed by Goyal [4, 5], Banarjee [6], and Goyal and Nebebe [7] implicitly assumed that the transportation cost is a part of the ordering cost. Ertogral et al. [8] suggested that this assumption is invalid as transportation cost can be affected by the routing decisions and the selected shipment size. They developed a shipment models based on two different discounting formats. The first describes transportation cost to be an all-unit-discount format while the second assumes it to be a function of the shipment lot size with overdeclaration option. Chung and Wee [9] noticed that using a single-order quantity with Just in Time multiple deliveries can significantly reduce the channel costs. However, the three-stage inventory model excluded the option of transporting discounts.

Unlike earlier research findings, in this paper, a multilevel inventory model is developed for procurement of outsourced components. It provides the outsourcing firms a tool to ensure competitive edge in the logistic performance. It diminishes all existing noncoordination, between various stages, involved in the logistic chain of an outsourcing firm. It encompasses the need to avoid shipment decisions biased to the transportation discount options, therefore, reemphasizing the significance of integrated inventory systems. The model also assimilates the formulation of inventory level at a stage where two different shipment policies are adopted resulting in two different frequencies of the inflow and outflow of the stock. The optimal values of these shipping frequencies are also achieved through this model. The model is flexible in deriving the optimal shipment sizes without any assumption on the holding costs. Another salient feature of this model exists in its solution methodology. The problem of the type defined above requires inspection of all feasible solutions for the determination of its optimal solution. However, with slight increase in the values of problem parameters, the search domain of the problem instance increases exponentially. This leads to enormous computational complexity which cannot be handled by deterministic approaches. Therefore, for such problems, various artificial intelligence-based stochastic search has been proposed in literature. These techniques are genetic algorithm [10, 11], ant colony optimization [12, 13], and particle swarm optimization (PSO) [14]. Owing to faster convergence, better exploration and exploitation abilities, and consistent performance in producing near-optimal results, this paper utilizes PSO as its basic search mechanism. PSO, first introduced by Kennedy and Eberhart [15], is inspired by the natural behavior of flocking birds. The movement of each particle of the swarm depends upon the particle's cognitive and social components. The cognitive component motivates the particle to attain the best position found by it so far, whereas the social component moves the particle toward the global optimum. However, to enhance the search capability of PSO, various improvements have been suggested in its basic structure. Further, Eberhart and Shi [16] showed that PSO is an effective optimizer, especially in large convoluted search spaces. Pandey et al. [17] proposed EPSO, an enhanced version of PSO, which provides the particles with additional information through a primitive component apart from the social and cognitive components used in the basic PSO. The model is solved by adopting EPSO strategy and deriving the optimal decisions to achieve the minimum total channel cost. It leads to better search for the near-optimal solution in reduced computational time.

Considering the aforementioned issues and existing research gap to address the outsourcing issues, an attempt has been made to develop a model to imitate the real-life scenario. Based on these premises, this research discusses the following objectives:

1. To obtain the optimal shipment frequencies for three-stage outsourcing
2. Use of evolutionary algorithm for deriving optimal shipment quantities
3. Incorporating exporting discount in the export size decision making
4. Inventory management of a system involving different frequencies of inflow and outflow of stock

To describe the above-mentioned features, the paper is divided into six sections. Following the introduction section, the rest of the paper is organized in following manner. Section 2 deals with the problem description and its assumptions. Section 3 presents the mathematical formulation of the problem while Section 4 details the solution methodology and its implementation on the model. In Section 5, the model is illustrated with a numerical example followed by the discussion of results in Section 6. In the last section, Section 7, conclusion has been portrayed.

2 Problem description

In daily life, we come across numerous products manufactured through assembly, in which some or many of them are outsourced to other collaborators within the same country (or to a different one) based on the production cost, quality, and abundance. As depicted in Fig. 1, components required for assembly of a product are shown with different geometrical shapes. Four square pieces, four cubes, three circular plates, a pair of disks, a pair of triangles, and a rhombus represent the various components, to be outsourced. These components are outsourced from different manufacturing units located in different countries. The components shown by cubes, circular plates, and the disks are outsourced to country X, rhombus to country Y, and the triangles to country Z, whereas components represented by the square pieces are manufactured at a location close to the assembly plant in the country of origin.

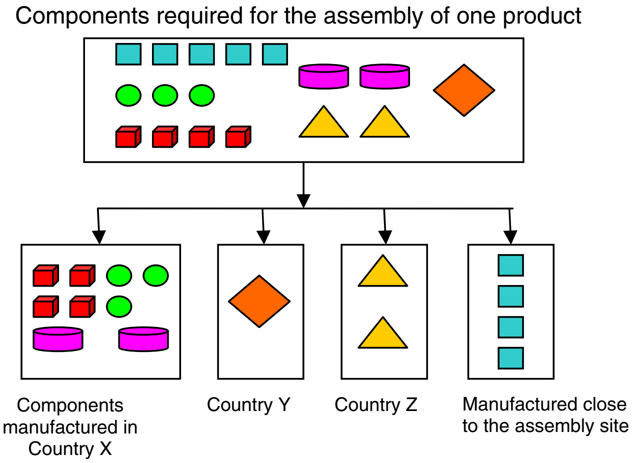


Fig. 1 Outsourcing strategy on component basis

Generally, procurement of the outsourced components involves three stages:

- Production of the components at the manufacturing units (MUs)
- The collection of the manufactured components at the exporting point (EP) or the collection center. This center refers to the best location in the country to export the components to the assembly plant
- Assembling the imported components at the assembly plant (AP)

A diagram outlining the different stages in the procurement of outsourced components from country X is depicted in Fig. 2. It presents the two shipment strategies that are adopted in these stages for successful completion of outsourcing. These are the inland shipment policy and the export shipment policy.

To illustrate the shipment policies, a term “set” is defined as the collection of components which are sufficient for assembly of one product. For example, for the product presented in Fig. 1, a set of components produced in country X is a collection of four cubes, three circular plates, and a pair of disks.

2.1 Inland shipment policy (MUs to EP)

It governs the shipment from the manufacturing units to the exporting point. Figure 2 exhibits that the three manufacturing units located in country X delivers the components to exporting point through inland shipment policy. Let Q_j be the demand for the manufacturing unit of component j during the cycle time T_c. This demand is fulfilled in “n” equal interval shipments to the EP as shown in Fig. 3.The first and last shipments received at the exporting point are denoted by q_{fs} and q_{ls}, respectively, while q_s is used to represent all other shipments.

The sizes of inland shipments are dependent on the total number of sets of components (Q_s) required in one cycle, as described below:

1. When the number of inland shipments n is a perfect multiplier of Q_s, then all the shipments will be of equal size (Q_s/n) irrespective of the difference in holding costs of the manufacturing units and the exporting point. Suppose, the number of sets (Q_s) to be shipped is 100 and the number of inland shipments is 10 (n=10), then all the shipment sizes would be equal to 10(Q_s/n).
2. When the number of inland shipments n is not a perfect multiplier of Q_s, there would be “$n-1$” shipments of same shipment size and one shipment of a different size. By intuition, we can infer that the production and shipment of odd size delivery in inland shipments should minimize the holding cost. In general, holding cost at any holding point in supply chain depends on many factors such as geographical region, labor cost, etc. For accounting odd shipment factor in our problem, we consider the following two conditions:

Fig. 2 Shipment policies for proposed model

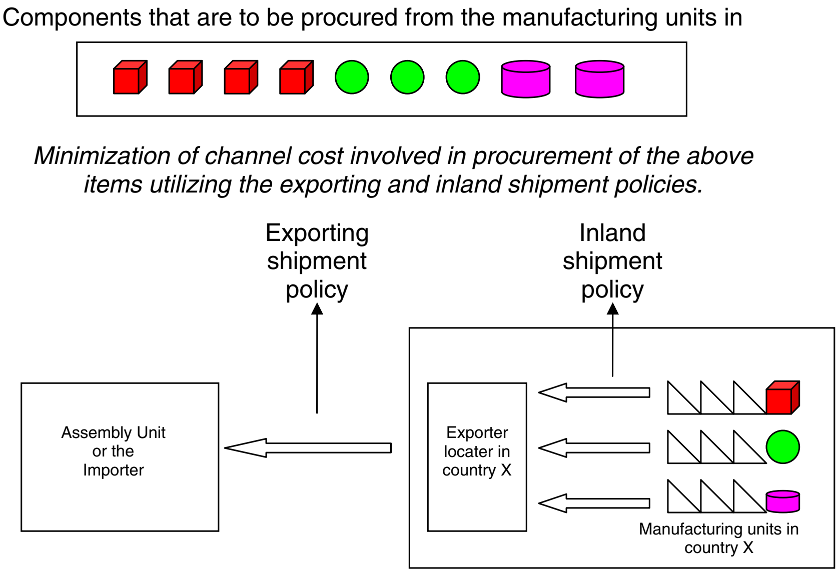
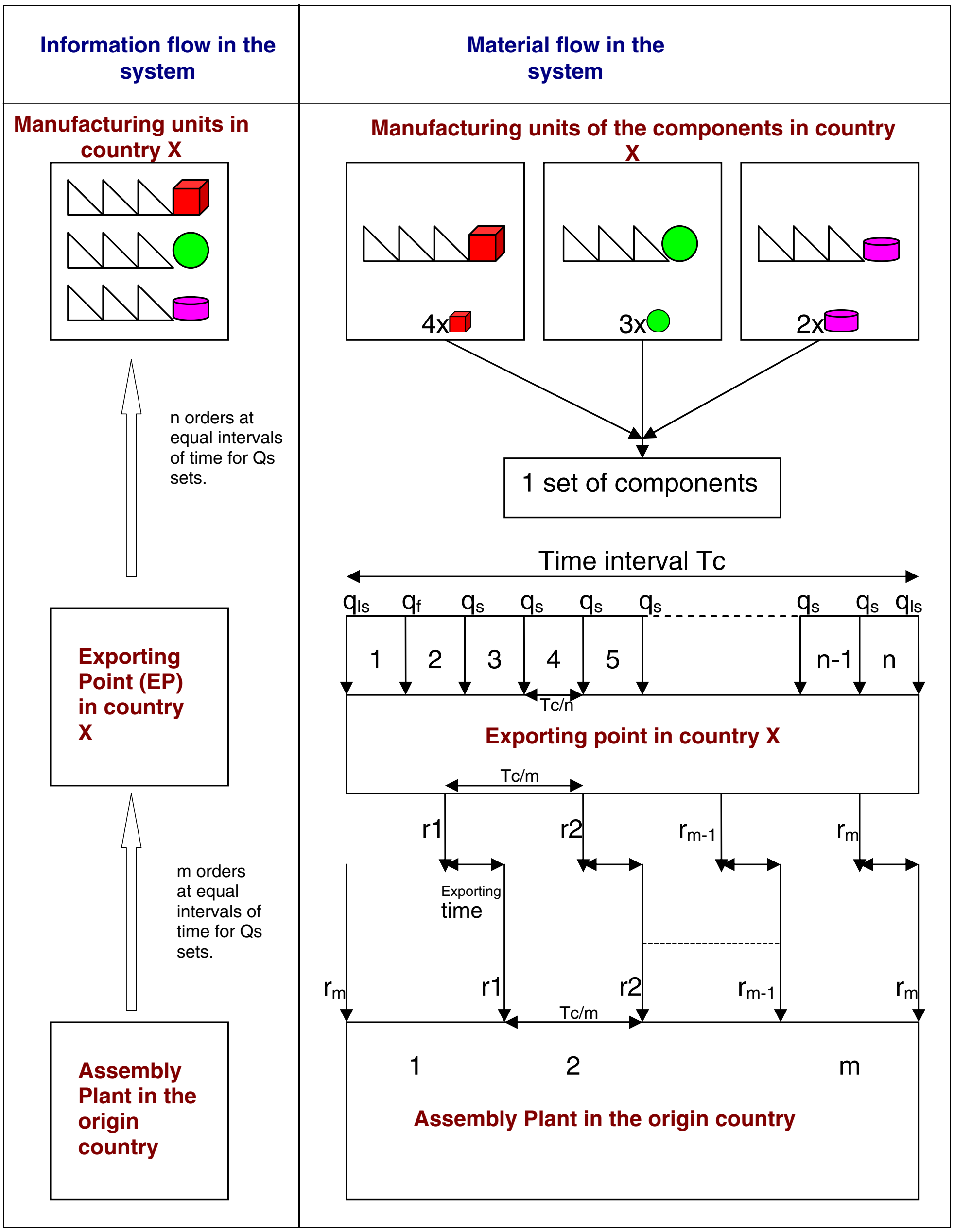


Fig. 3 Material and information flow



(a) When the holding cost of the manufacturers is greater than the holding cost of the exporter, less quantity should be stored at the manufacturer, which makes the last shipment odd, i.e., smaller shipment size will be the last one. This is illustrated in Table 1 where the number of shipments (n) is 10 and the number of sets (Q_s) to be shipped to the exporting point is 99 and 105 then last shipments have the least sizes 9 and 6, respectively.

(b) When the holding cost of the manufacturer is less than the holding cost of the exporter, inventory should be held at the manufacturer rather than the exporter which makes the first shipment (q_{fs}) to be less than any other shipment during the time interval T_c. It can be observed that the lower shipment sizes 9 and 6 are now as the first shipment sizes when the number of sets (Q_s) is 99 and 105, respectively.

2.2 Export shipment policy (EP to AP)

This policy governs the shipments from the exporting point to the assembly plant. The components received at the EP are transported to the assembly plant in "m" shipments of shipment sizes r_1, r_2, r_3, …, r_m (see Fig. 3) at regular intervals of time (t). The shipment sizes of r_1, r_2, r_3, …, r_m are decided as follows:

1. No export size can exceed the stock available with the exporter/collection center.

Table 1 Variation of inland shipment sizes with demand during time T_c

Number of shipments (n)	Quantity to be shipped to collection center during T_c (Q_s)	Quantity in first shipment (q_{fs})	Quantity in each of second to ninth shipments (q_s)	Quantity in tenth shipment (q_{ls})
Case 1: when the average holding cost of the manufacturing units is greater than the average holding costs at the exporting point				
10	100	10	10	10, no odd set
10	99	10	10	9, odd set
10	105	11	11	6, odd set
Case 2: when the average holding cost of the manufacturing units is lesser than the average holding costs at the exporting point				
10	100	10, no odd set	10	10
10	99	9, odd set	10	10
10	105	6, odd set	11	11

2. The minimum size of an export is the size which could avoid the shortage at the assembly plant until the next export arrives.

Apart from the aforesaid issues, effective utilization of the discount scheme also plays an important role in curtailing the overall cost. For the scheme considered here for containerization transport, the exporting cost is constant for any shipment size in a given range and the discount is based on the number of containers hired rather than the number of units in a container. The exporting discount can be illustrated with the help of Fig. 4. There is no discount offered for the shipment range that requires one container. However, as the shipment range increases, discount also increases up to a limit. After this point, it remains constant for a greater shipment ranges.

2.3 Notations and assumptions

The following notation and assumptions are made in the development of the inventory-based multilevel outsourcing model:

1. C_{oc} and C_{oa} are the ordering costs for the EP and the AP for each shipment, respectively.

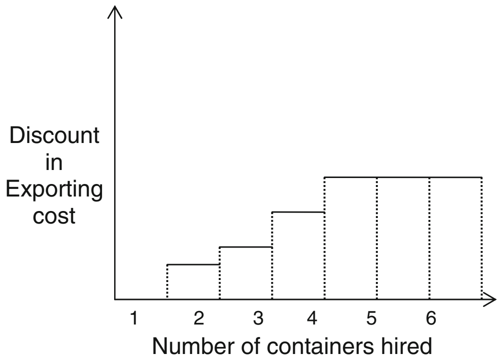


Fig. 4 Discount model presentation

2. d_j is the number of the component j required for the assembly of one product.
3. D is the demand rate of the product at the assembly plant ($D=Q\ /\ T_c$).
4. H_a is the holding costs at the assembly plant expressed as cost per unit per cycle time.
5. H_c is the holding costs at the exporting point expressed as cost per unit per cycle time.
6. H_{pj} is the holding cost for the manufacturing unit of the component j for a unit inventory/unit time
7. P_j is the production rate of the component j where $j=1, 2, 3, \ldots, w$ and w is number of manufacturing units.
8. Q is the quantity of products to be assembled during the time period T_c.
9. Q_j is the quantity of component j required for the assembly of Q products ($Q_j=Q\times d_j$).
10. Q_s is the total quantity of sets required for the assembly of Q products during the time period T_c ($Q_s=q_{fs}+(n-2)\times q_s+q_{ls}$).
11. T_c is the cycle time.
12. u is the delivery time from the exporting point to the assembly plant.

2.4 Decision variables

The decision variables include the following:

1. f_j is the fraction in the k_j time interval T at which the production of the component j stops during the time T_c.
2. k_j is the index of the time interval T in which the production of the component j stops during the time T_c.
3. m is the number of exports made from the exporting point to the assembly plant.
4. n is the number of inland shipments taken place between the manufacturing units and the exporting point during the time interval T_c.

5. q_{fj} is the quantity of components j shipped to the exporting point from the manufacturing unit in the first shipment of the time interval T_{c}.
6. q_j is the quantity of components j shipped to the exporting point from the manufacturing unit in the shipments other than the first and the last shipment of the time interval T_{c}.
7. q_{lj} is the quantity of components j shipped to the exporting point from the manufacturing unit in the last shipment of the time interval T_{c}.
8. q_{fs} is the total quantity of sets received by the exporter from all manufacturing units in the first shipment of the time interval $T_{\mathrm{c}}\left(q_{\mathrm{fs}}=\sum_{j=1}^{w} q_{\mathrm{fj}}\right)$.
9. q_{s} is the total quantity of sets received at the exporting point from all manufacturing units in the shipments other than the first and the last shipment of the time interval $T_{\mathrm{c}}\left(q_{\mathrm{s}}=\sum_{j=1}^{w} q_j\right)$
10. q_{ls} is the total quantity of sets of components received by the exporter from all manufacturing units in the last shipment of the time interval $T_{\mathrm{c}}\left(q_{\mathrm{ls}}=\sum_{j=1}^{w} q_{\mathrm{lj}}\right)$
11. r_b is the size of the bth export from the collection center/exporter to the assembly plant, where $b=1, 2, 3, \ldots, m$.
12. t is the time interval between two successive shipments made from the exporting point to the assembly plant during the time interval T_{c} ($t=T_{\mathrm{c}}/m$).
13. T is the time interval between two successive shipments made by the manufacturing units to the collection center ($T=T_{\mathrm{c}}/n$).

The following assumptions have been considered for realization of the proposed model:

1. The production rate of each of the components is greater than demand rate of the components.
2. The consumption rate of the components at the assembly plant is continuous and constant. Only the required numbers of components are produced by the manufacturing units that is transported to and consumed by the assembly plant during the time period T_{c}.
3. No shortage of components occurs at the assembly plant.
4. In order to avoid zero lot size shipment, the delivery frequency is less than or equal to the inland shipping frequency. ($m \leq n$)
5. Each component can be made at only one manufacturing unit.
6. The inland shipment policy assumes instantaneous replenishment and the transportation time is constant; hence, no time delay and transportation cost have been considered.
7. No damages or defects occur in the components at any stage of the procurement process.
8. Shipment of different types of components from MUs to exporting point follow the same ration pattern and all have n times of shipments.
9. In transit inventory has not been considered.

3 Mathematical formulation

The proposed inventory-based multilevel outsourcing model (IMOM) assimilates different costs involved in the procurement of the outsourced components. With the flow of demand and order, inventory at each stage changes over the period and affects the total channel cost. The total channel cost is expressed as the sum of the following costs:

1. The total average holding costs of the manufacturers, collector/exporter, and the assembly plant during the time period T_{c}
2. Setup costs by the manufacturers and ordering cost of the exporting point and the assembly plant
3. Exporting cost of the components from the collection center to the assembly unit

These costs are delineated in the following sections:

3.1 Total average inventory holding costs

The total average holding cost is calculated as the sum of the average holding costs at the manufacturing units, collection center, and the assembly plant. Figures 5, 6, and 7 illustrate the inventory flow pattern of a manufacturing unit, the exporting point, and the assembly plant, respectively. The average time-weighted inventory is determined as the area under the curve for each of the inventory flow diagram. The average holding cost for the time period T_{c} is expressed as follows:

$$\begin{aligned}&\text{Average holding cost for the time period } T_{\mathrm{c}}\\&=\text{Average time weighted inventory}\\&\quad\times T_{\mathrm{c}}\times\text{holding cost per unit per } T_{\mathrm{c}}.\end{aligned}$$

The proposed inventory model and the associated cost for the manufacturing units, collection center, and the assemble plant is delineated as follows:

3.1.1 Total average inventory holding cost at the MUs

The total average inventory holding costs of the manufacturing units is the sum of the holding costs of all units. Figure 5 presents the inventory flow diagram at one of the

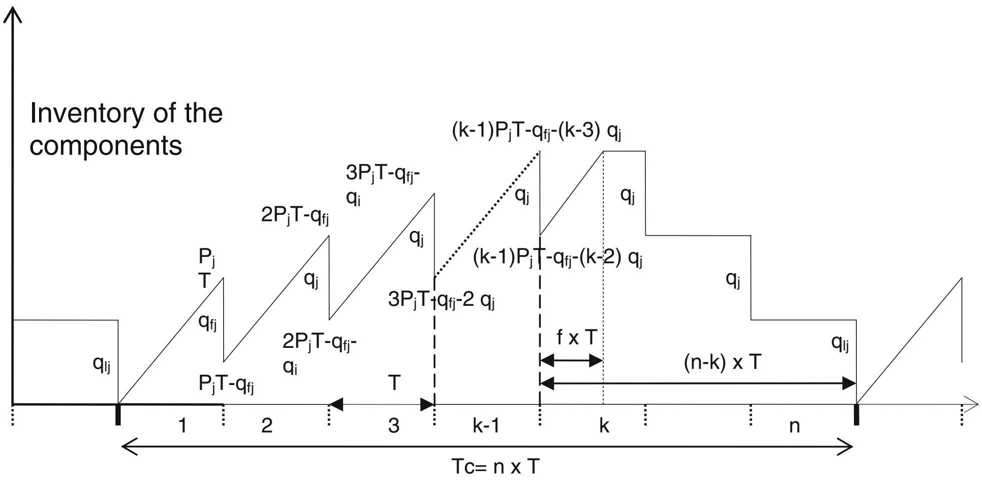


Fig. 5 Inventory flow for a manufacturing unit

manufacturing units. The inventory at the start of each cycle is assumed to be zero. The production rate for a component j is P_j. Therefore, in a time interval (T), the inventory of component j increases by $P_j \times T$. At the end of the first interval of T_c, the first shipment of size q_{fj} takes place. Therefore, the net inventory at the end of the first time interval is $P_jT - q_{fj}$ (see Fig. 5). Similar pattern is followed up to ($k_j - 1$) intervals. The demand of component j is met in the k_jth interval and the production stops. Thus, this interval contains two phases namely the production phase and the nonproduction phase. The time length of the production phase is denoted by $f_j \times T$, where f_j is a fractional value. While the nonproduction phase is of the length $(1-f_j) \times T$ as shown in Fig. 5. For the remaining $(n-k_j)$ intervals, the inventory level remains constant and decreases by the shipment made at the end of the interval. At the end of the nth interval, the last shipment q_{lj} is made and the inventory becomes zero for the start of the next cycle.

The average time-weighted inventory of manufacturing unit of component j during the time interval is as follows:

1. First time interval T of $T_c = P_jT^2/2$
2. Second time interval T of $T_c = \left[P_jT^2/2 + \left(P_jT - q_{fj}\right)T\right]$
3. Third time interval T of $T_c = \left[P_jT^2/2 + \left(2P_jT - q_{fj} - q_j\right)T\right]$
4. $(k_j - 1)$th time interval T of $T_c = \left\{P_jT^2/2 + \left[\left(k_j - 2\right)P_jT - q_{fj} - \left(k_j - 3\right)q_j)\right]T\right\}$.

As described earlier, in the k_jth time interval, only f_j fractional part is the production phase. This generates the two cases of inventory:

1. Production phase (f_jT) of k_jth interval of $T_c = P_j\left(f_jT\right)^2/2 + \left[\left(k_j - 1\right)P_jT - q_{fj} - \left(k_j - 2\right)q_j\right]f_jT$
2. Nonproduction phase $(1-f_j)T$ of k_jth interval $= \left[\left(k_j - 1\right)P_jT - q_{fj} - \left(k_j - 2\right)q_j\right]f_jP_jT\right]\left(1 - f_jT\right)$.

Now, the inventory of the remaining nonproduction phase pf ($n-k_j$th) T intervals of $T_c = 1 + 2 + 3 + \ldots + \left(n - k_j - 1\right)q_jT + \left(n - k_j\right)q_jT$ and total time-weighted inventory during the time T_c of a manufacturing unit of component

$$j = AP_jT^2 + Bq_{fj}T + Cq_jT + Dq_{lj}T \qquad (1)$$

where $A = \left(k_j^2 - \left(1 - f_j\right)^2\right)/2$, $B = 1 - k_j$, $C = \left[\left(n - k_j\right)\left(n - k_j - 1\right) - \left(k_j - 1\right)\left(k_j - 2\right)\right]/2$, $D = \left(n - k_j\right)$, and $T = T_c/n$.

Thus, total time-weighted inventory during the time T_c of all the manufacturing units comes out to be

$$= \sum_{j=1}^{w} AP_jT^2 + Bq_{fj}T + Cq_jT + Dq_{lj}T \qquad (2)$$

where w is the total number of manufacturing units of various outsourced components. Hence, the total holding cost during the time T_c of all the manufacturing units comes out to be

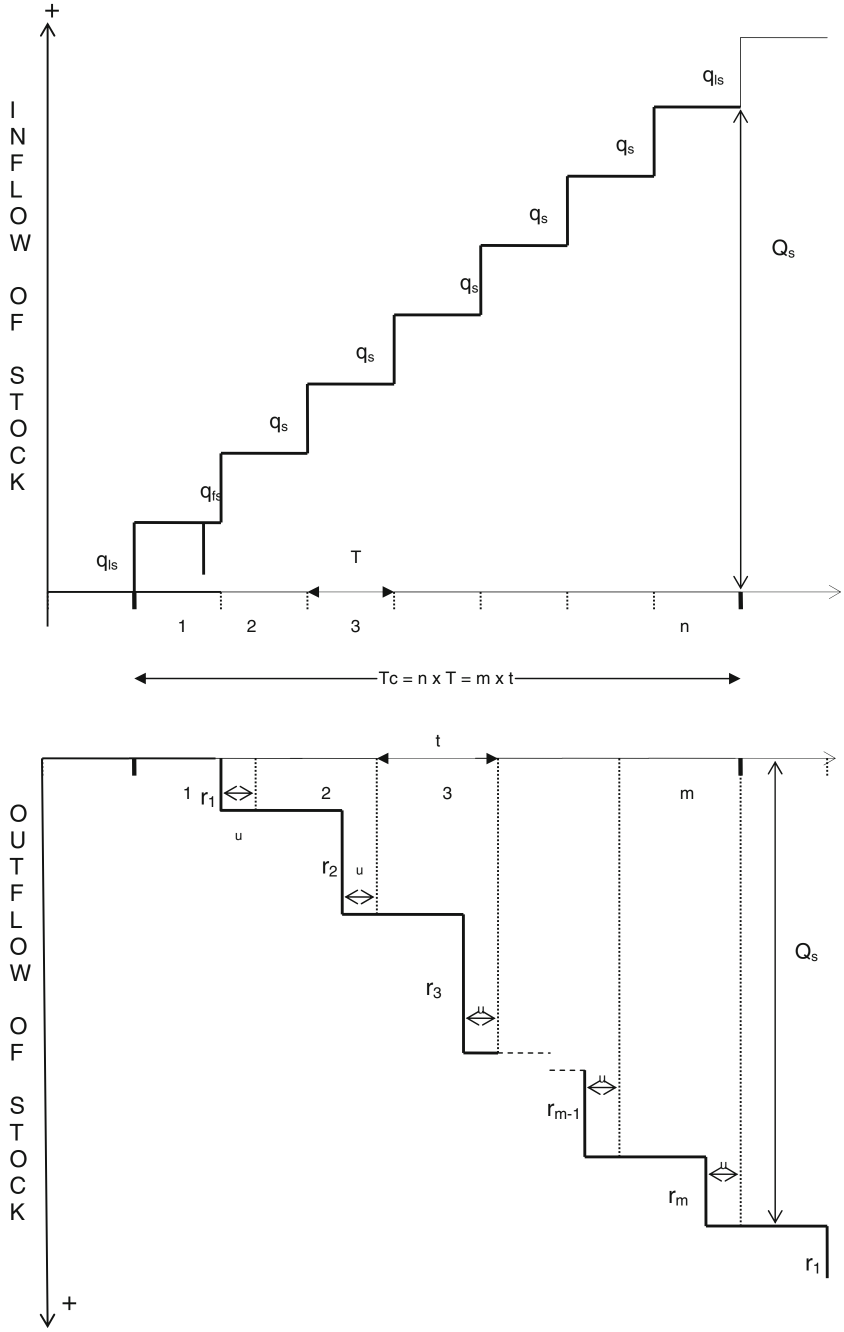
$$= \sum_{j=1}^{w} \left[\left(AP_jT^2 + Bq_{fj}T + Cq_jT + Dq_{lj}T\right)T_cH_{pj}\right]. \qquad (3)$$

3.1.2 Total average inventory holding cost at the EP

The collection center or the exporting point is the midstage of the IMOM. At this level, inflow and outflow of the components make inventory management-related decisions a crucial one. A careful investigation as well as monitoring is required at this stage. Figure 6 demonstrates the inflow and outflow of inventory of the components at the collection center. The components arrival frequency is greater than its exporting frequency (i.e., $m \leq n$):

Average time weighted inventory at the collection center
= time weighted ([average inflow of inventory]
− [average outflow of inventory]) (4)

Fig. 6 Inventory flow for the collector/exporter



$$\begin{aligned}
&= [q_{ls}T_c + (n-1)q_{fs}T_c + (1+2+3+\ldots+n-2)q_s - T] - [r_1(m-1)t + r_2(m-2)t + \ldots + r_{m-1}(1)t + Q_s u] \\
&= [(q_{ls} + (n-1)q_{fs})T_c + (n-1)(n-2)q_s T/2] - [r_1(m-1)t + r_2(m-2)t + \ldots + r_{m-1}(1)t + Q_s u] \qquad (5) \\
&= [(q_{ls} + (n-1)q_{fs}/n)T_c + (n-1)(n-2)q_s T_c/2n] - [(r_1(m-1) + r_2(m-2) + \ldots. + r_{m-1})T_c/m + Q_s u]
\end{aligned}$$

Average holding cost of the collection center during time T_c

$$= \{[(q_{ls} + (n-1)q_{fs})T_c + (n-1)(n-2)q_s T_c/2n] - [(r_1(m-1) + r_2(m-2) + \ldots + r_{m-1})T_c/m + Q_s u]\}T_c H_c \qquad (6)$$

where n is the number of inland shipments made during time interval T_c, m is the number of export made during time interval T_c, q_{fs} and q_{ls} are the number of set of components received at collection center at the end of first and last time intervals of T_c,

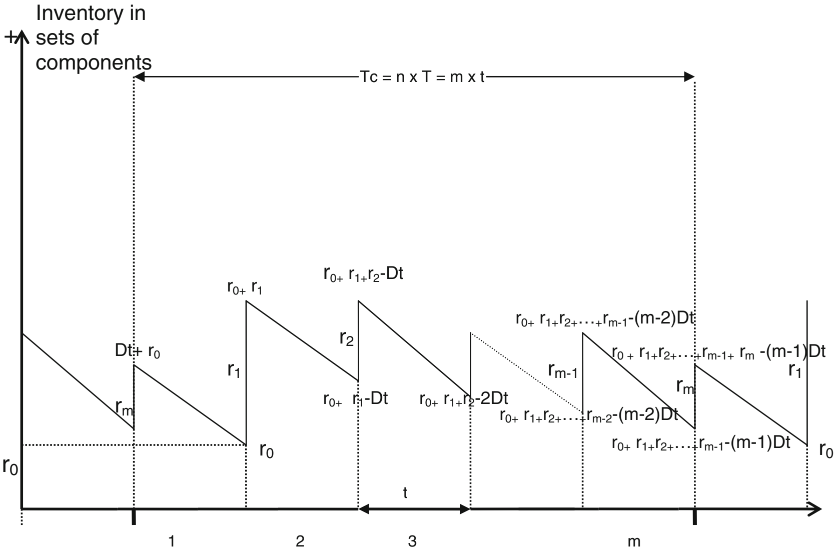


Fig. 7 Inventory flow for the assembly plant

and q_s is the number of set of components received at the collection center in any other shipment during time interval T_c.

3.1.3 Total average inventory holding cost at the AP

The components shipped from the exporting point are consumed at a constant rate ($D=Q\ /\ T_c$) at the assembly plant. The assembly plant maintains a safety stock of r_0 (sets of the components). Figure 7 depicts that the assembly plant inventory increases by r_b at the end of bth time interval t during the time period T_c and decreases by an amount of $D\times t$ during the time interval t. It is due to the fact that the assembly plant consumes the components at a constant rate D and receives a shipment r_b at the end of bth time interval t from the assembly plant. Since the assembly plant is supplied with the required number of components in every cycle, the inventory returns to the initial level r_0 at the arrival of the first shipment of the following cycle.

The area under the plot in Fig. 7 gives the time-weighted inventory at the assembly plant. For component j, during different time interval, it is found to be as follows:

1. First time interval t of $T_c = Dt^2/2 + r_0t$
2. Second time interval t of $T_c = Dt^2/2 + (r_0t + r_1t - Dt^2)$
3. Third time interval t of $T_c = Dt^2/2 + (r_0t + r_1t + r_2t - 2Dt^2)$
4. $(m{-}1)$th time interval t of $T_c = Dt^2/2 + (r_0t + r_1t + \ldots + r_{m-2}t - (m-2)Dt^2)$
5. mth time interval t of $T_c = Dt^2/2 + (r_0t + r_1t + \ldots + r_{m-1}t - (m-1)Dt^2)$

Total time-weighted inventory during the time T_c of the assembly unit/importer (the sum of all the above equations derived for m intervals)

$$= [mr_0 + (m-1)r_1 + (m-2)r_2 + \ldots + 2r_{m-2} + r_{m-1}]t - \left(m^2 - 2m\right)Dt^2/2. \quad (7)$$

Average holding cost at the assembly unit during time T_c

$$= \left\{[mr_0 + (m-1)r_1 + (m-2)r_2 + \ldots + 2r_{m-2} + r_{m-1}]t - \left(m^2 - 2m\right)Dt^2/2\right\}T_cH_a. \quad (8)$$

The total average holding cost is the sum of the average holding costs of the manufacturing unit, collection center, and the assembly plant (i.e., sum of Eqs. 3, 6, and 8).

Hence, the total average holding cost comes out to be

$$\begin{aligned} = &\sum_{j=1}^{w}\left[\left(APT^2 + Bq_{fj}T + Cq_jT + Dq_{fj}T\right)T_cH_{pj}\right] + \{[(q_{ls} + (n-1)q_{fs})T_c + (n-1)(n-2)q_sT_c/2n] \\ &-[(r_1(m-1) + r_2(m-2) + \ldots + r_{m-1})T_c/m + Q_su]\}T_cH_c \\ &+\left\{[mr_0 + (m-1)r_1 + (m-2)r_2 + \ldots + 2r_{m-2} + r_{m-1}]t - (m^2 - 2m)Dt^2/2\right\}T_cH_a. \end{aligned} \quad (9)$$

3.2 Setup costs and the ordering costs

In addition to the inventory holding costs, proposed model also encapsulates the setup costs and the ordering costs incurred by the exporter and the assembly plant. The total setup cost is the sum of the set-up costs of all the manufacturing units (S_0). The ordering cost for each order is assumed to be constant for both the exporter and the assembly units. The ordering cost is found to be the product of the number of orders and the cost of each order:

Total setup costs of all the manufacturing units

$$S_0 = \sum_{j=1}^{w} S_j \tag{10}$$

Total ordering costs of the collection center during

$$T_c = n \times C_{oc} \tag{11}$$

Total ordering costs of the assembly unit during

$$T_c = m \times C_{oa} \tag{12}$$

3.3 Exporting cost

Exporting cost is also considered to be a major cost in this model. The exporting cost for each shipment is calculated on the basis of a discounting system shown in Fig. 4.

Total exporting cost for the m shipments during the time

$$T_c = \sum_{b=1}^{m} (r_b \times C(r_b)) \tag{13}$$

where r_b is the size of the bth export to the assembly plant where b=1, 2, 3,…, m and $C(r_b)$ is the exporting cost of the r_b shipment.

Our aim is to minimize the total channel cost involved in the procurement of the outsourced components that involves inventory holding cost, setup cost, ordering cost, and the exporting cost (i.e., aggregating Eqs. 9, 10, 11, 12, and 13). The assumption of constant demand eliminates the role of setup costs in the minimization of total channel cost. Thus, the objective function can be expressed as follows:

Minimization of total channel cost
= Minimization (Total average holding cost units + Total ordering costs of the collection center + Total ordering costs of the assembly unit + Totalexporting cost) + Total setup costs of the manufacturing

$$\begin{aligned} &= \text{Minimization of } \left\{ \sum_{j=1}^{w} \left[\left(APT^2 + Bq_{fj}T + Cq_jT + Dq_{fj}T \right) T_c H_{pj} \right] + \{[(q_{ls} + (n-1)q_{fs})T_c \right. \\ &+(n-1)(n-2)q_s T_c/2n] - [(mr_m + r_1(m-1) + r_2(m-2) + \ldots + r_{m-1})T_c/m + Q_s u]\} T_c H_c + \{[mr_0 + (m-1)r_1 \\ &\left. +(m-2)r_2 + \ldots + 2r_{m-2} + r_{m-1}]t - (m^2-2m)Dt^2/2\} T_c H_a + n \times C_{oc} + m \times C_{oa} + \sum_{i=1}^{m} (r_i \times C_{r_i}) \right\} + \sum_{j=1}^{w} S_j. \end{aligned} \tag{14}$$

3.4 Constraints

The model is formulated to map the most of the real-life outsourcing inventory problem; there exist certain constraints associated with the formulation that are required to be taken care. The objective function formulated in Eq. 14 is subjected to the following constraints:

- The delivery frequency (m) is less than or equal to the inland shipping frequency (n):

$$m \leq n. \tag{15}$$

- The total size of all the inland shipments and the total size of all the exports should be equal to the demand during the time period T_c, i.e.,

$$q_1 + q_2 + q_3 + \ldots + q_n = q_{fs} + (n-2)q_s + q_{ls} = Q_s \tag{16}$$

$$r_1 + r_2 + r_3 + \ldots + r_{m-1} + r_m = Q_s. \tag{17}$$

This is due to the fact that the during the time period T_c, the exporter receives Q_s set of components and exports the same quantity.

- Any delivery size cannot exceed the stock available with the exporter/collection center

$$r_b \leq q_{ls} + q_{fs} + (Z_b - 1)q_s - (r_1 + r_2 + r_3 + \ldots + r_{b-1}) \tag{18}$$

where r_b≤(initial inventory+(stock received−stock exported) before the r_b shipment), initial inventory at the starting of the time period T_c=q_{ls}, stock received before the r_b shipment=q_{fs}+(Z_b−1)q_s, and stock exported before the r_b shipment=$r_1+r_2+r_3+\ldots+r_{b-1}$ where b=1, 2, 3, …, m and Z_b is the integer value which gives the number of inland shipments made before the bth export from the collection center to the assembly plant, i.e., Z_b = integer part of $\left(\frac{n}{m}\times b\right)$.

- The minimum size of an export is the size which could avoid the shortage at the assembly plant until the next export arrives. From Fig. 7, we can say that to avoid shortage at the assembly plant, the following conditions should be satisfied: $r_1 \geq Dt - r_0$, $r_2 \geq 2Dt - (r_0 + r_1)$, $r_3 \geq 3Dt - (r_0 + r_1 + r_2)$, and $r_{m-1} \geq (m-1)Dt - (r_0 + r_1 + r_2 \ldots + r_{m-2})$. But from Eq. 17, $r_m = mDt - (r_1 + r_2 +, \ldots, + r_{m-2} + r_{m-1})$; hence, the general equation for the export shipment size comes out to be

$$r_b \geq (D \times t)(b) - (r_0 + r_1 + r_2 + r_3 + \ldots + r_{b-1}) \quad (19)$$

  where b=1, 2, 3, …, $m-1$.

Considering the formulation and associated constraints, it is imperative to solve with a model that can search the solution space with due consideration of problem sensitivity.

4 Solution methodology

The proposed model solves the multivariate and computationally complex problem, as the search space is large and sensitive to the changes in the variable values. For example, let the range of inland shipments (n) lies between 1 and 10. For each n, there are 10 possibilities of m as $m \leq n$. Moreover, for each value of m, there are different possible combinations of export shipments r_1, r_2, r_3, …, r_m. Since cost associated with each of these is dissimilar, it is expensive in terms of computational cost to check every possible combination of the decision variables. The other complicacy is the number of decision variables (see Table 2). The total number of decision variables varies with the number of exports (i.e., m+2).

Since the deterministic methods like dynamic programming, minimal cut, and branch and bound algorithm are not suitable for solving this model, nature-inspired evolutionary algorithm is used to obtain the near-optimal solution. Due to the global and local exploration abilities, faster convergence, and consistency in the performance of PSO, the proposed model utilizes its variant EPSO. PSO, first introduced by Kennedy and Eberhart [14] is inspired by the natural behavior of flocking birds. The particles move to new positions with velocities guided by the knowledge of the best position attained by the swarm known as the global best and also the best position attained by the particle itself called as the particle best by the following equations:

Table 2 Variation in the number of variables with number of exports

Number of exports during time T_c (m)	Decision variables	Number of decision variables
3	n, m, r_1, r_2, r_3	5
5	n, m, r_1, r_2, r_3, r_4, r_5	7

$$\vec{V}_i(t+1) = w\vec{V}_i(t) + c_1 r_c\left(\vec{P}_{ib} - \vec{X}_i(t)\right) + c_2 r_s\left(\vec{P}_g - \vec{X}_i(t)\right) \quad (20)$$

$$\vec{X}_i(t+1) = \vec{X}_i(t) + \vec{V}_i(t+1) \quad (21)$$

where $\vec{X}_i(t)$ and $\vec{V}_i(t)$ represent the position and velocity of a particle i at time t, w is the inertia weight of the initial velocity, c_1 and c_2 are cognitive and social acceleration constants, r_c=[0, 1] and r_s=[0, 1] are random parameters with uniform distributions, $\vec{P}_{ib}$ refers to the particle's best position, and $\vec{P}_g$ refers to the global best position of the swarm.

In addition to the cognitive and social components used in Eq. 20, a primitive component was introduced by Pandey et al. [17] in EPSO. Its mathematical representation of velocity update can be given as follows:

$$\vec{V}_i(t+1) = w\vec{V}_i(t) + c_1 r_c\left(\vec{P}_{ib} - \vec{X}_i(t)\right) + c_2 r_s\left(\vec{P}_g - \vec{X}_i(t)\right) + c_3 r_n\left(\vec{P}_g - \vec{P}_{ib}\right) \quad (22)$$

where

$$w = (w_i - w_f) \times \frac{(I - E)}{I} + w_f, \quad (23)$$

$$c_1 = (c_{1f} - c_{1i}) \times \frac{E}{I} + c_{1i}, \quad (24)$$

$$c_2 = (c_{2f} - c_{2i}) \times \tfrac{E}{I} + c_{2i}, \quad c_3 = 0.5. \quad (25)$$

w_i and w_f are the initial and final inertia weights, I is the total number of iterations, and E is the current iteration. Shi and Eberhart [15] suggested that the optimal solution can be improved when w_i and w_f are taken as 0.9 and 0.4, respectively. The value for c_1 changes from (c_{1i}) 2.5 to (c_{1f}) 0.5 whereas c_2 changes from (c_{2i}) 0.5 to (c_{2f}) 2.5.

In the proposed model, EPSO is used to determine the best export shipment policy for a given number of inland shipments (n) and exports (m). The export shipment policies are used as the position vectors of the particles of the swarm. Each particle contains m bits to represent the shipments sizes of the all the exports during the cycle time T_c. To obtain the best export shipment policy for a given number of inland shipments and exports, initial population containing particles of feasible export policies is generated. The particle with the least total channel cost is said to have the maximum fitness value. The particles move to new positions with the velocities governed by Eq. 21; infeasible generated solutions are replaced by the nearest feasible ones.

The overall solution methodology for the outsourcing problem is described in the following steps:

Step 1: Generate feasible values of number of inland shipments (n) and exports (m).
Step 2: Derive the inland shipment sizes (q_{fs}, q_s, and q_{ls}) as per the inland shipment policy illustrated in Table 1.
Step 3: For population initiation, for each value of n, m initialize particles/solutions representing export shipment policies consist of $r_1, r_2, r_3, \ldots, r_m$.
Step 4: For total channel cost values, total channel cost for each combination of n, m, and $r_1, r_2, r_3, \ldots, r_m$ is calculated using Eq. 14.
Step 5: For EPSO, solution related to each set is determined and stored in the repository.
Step 6: For ranking of the solution, at termination, the solutions stored in the repository are ranked according to the minimum cost criteria. The solution with least cost is given rank 1 and obtained as the best solution.

The steps proposed above are illustrated with the help of a flow diagram shown in the Fig. 8. A numerical example is solved in the following section to present the application of the proposed methodology.

5 Example

The proposed model is delineated by simulating a case of car manufacturer. This leading manufacturer is denoted as ABC company. It assembles 300 special cars every month. Three components namely the power steering pump, driving lights, and clutch kits are procured from country X. Each car assembly requires one power steering pump, a pair of driving lights, and four sets of clutch kits. A maximum of one inland shipment can take place in a day. The inputs for the calculation of total channel cost are provided as follows:

1. Cycle time T_c=30 days
2. Number of sets of components required Q_s=300 sets
3. Production rate of power steering pump P_1=20/day
4. Production rate of driving lights P_2=30/day
5. Production rate of clutch kits P_3=50/day
6. Number of components power steering pumps to be produced/month Q_1=1×300=300
7. Number of components driving lights to be produced/month Q_2=2×300=600
8. Number of components clutch kits to be produced/month Q_3=4×300=1,200
9. Transportation time from exporting point to assembly plant u=20 h (constant)
10. Safety stock at the assembly unit=three sets.

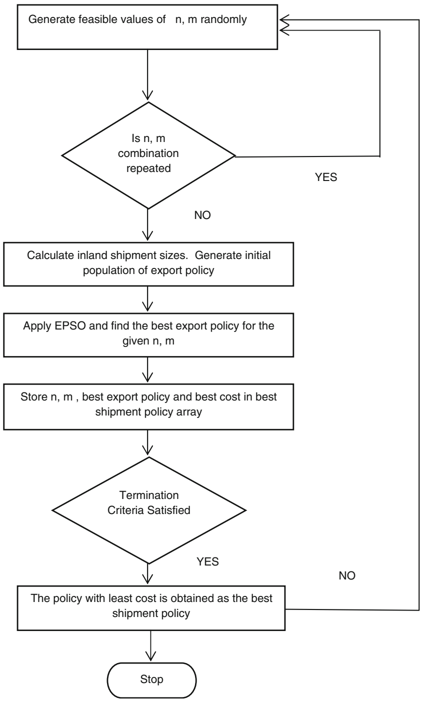


Fig. 8 Flow diagram for the solution methodology

Other additional input costs used in the example are given in Table 3. A program is written to solve the problem utilizing technical computing language tool, Matlab (version 7.1.0.246 R (14) service pack 3). The optimum number of n and m is determined along with the shipment sizes. Detailed analysis of the results has been made afterward.

6 Results and discussion

The optimal shipment policy is based on the total channel cost, involving inventory holding costs, setup costs, ordering costs, and the exporting costs. The best 25 particles exhibiting the 25 best export strategies are selected and shown in Table 4 along with the total channel cost.

The particles are ranked on the basis of minimum channel cost. The strategy with the least total channel cost is ranked

Table 3 Associated cost components

Name of the component	Component 1, power steering	Component 2, driving lights	Component 3, clutch kits	One set of components
Number required for 1 product	1 (d1=1)	2 (d2=2)	4 (d3=4)	Total 7 components/set
Holding costs				
Holding cost/component/month at the manufacturing unit	$H_{p1}=\$8$	$H_{p2}=\$6$	$H_{p3}=\$10$	$\$8+\$12+\$40=\$60$
Holding cost/component/month at the exporting center	\$5	\$3	\$7	$H_c=\$30$
Holding cost/component/month at the assembly unit	\$12	\$10	\$16	$H_a=\$100$
Setup and ordering costs				
Total setup cost incurred by all the manufacturing units in a period of 1 month($S_0=\sum_{j=1}^{w} S_j$)				$S_0=\$500$
Ordering cost for the exporting center per order				$C_{oc}=\$50$
Ordering cost for the assembly unit per order				$C_{oa}=\$80$
Exporting cost				
Number of component sets to be exported in the *x*th (r_x)	Containers required	Cost (without discount) (\$)	Discount (\$)	Exporting cost (\$)
$0<r_x\leq 50$	1	400	0	400
$50<r_x\leq 100$	2	800	100	700
$100<r_x\leq 150$	3	1,200	250	950
$150<r_x\leq 200$	4	1,600	500	1,100
$200<r_x\leq 250$	5	2,000	500	1,500
$250<r_x\leq 300$	6	2400	500	1,900
$300<r_x\leq 350$	7	2,800	500	2,300

1 and considered as the best solution for the given problem. Therefore, from Table 4, it can be observed that the optimal shipment policy for ABC company should consist of the eight inland shipments ($n=8$) where the first seven shipments are of size 42 whereas the last shipment consists of six sets. Further, this policy suggests that exporting frequency should be the eight per month with shipment sizes of 19, 38, 35, 42, 74, 48, 44 consecutively.

To study the effect of the shipment policy decisions on the total channel cost, we analyze the variation of total channel cost with the decision variables n, m, and $r_1, r_2, r_3, \ldots, r_m$.

6.1 Effect of the number of inland shipments (n) on the total channel cost

It is interesting to uncover the variation of total channel cost with the number of inland shipments. The pattern of the total channel cost involves inventory holding costs, ordering costs, and exporting cost for different combination of number of inland shipments (n) and number of exports (m). In Fig. 9, one such pattern is shown to explain the effect of the number of inland shipments n on different costs for fixed number of exports ($m=6$) and corresponding best export polices.

When number of exports is fixed (six in this case), the ordering cost for the importer/assembly unit is unaffected by the number of inland shipments. While the ordering cost for the exporter increases linearly with increase in the number of inland shipments (n). The inland shipment policy did not have much effect on the exporting cost as the number of exports (m) is fixed. However, slight variation occurred due to the inventory inflow at the exporting point. The major costs that contributed to the total channel cost are the net holding costs. In the case of ABC company, holding cost at the assembly plant is predominant over the other holding costs as shown in Table 3. The total inventory holding cost curve is the summation of holding costs at the manufacturing units, exporting point, and the assembly plant. Thus, the decision of the number of inland shipments is made to reduce the net inventory holding costs to have the minimum total channel cost.

6.2 Effect of the number of exports (m) on the total channel cost

Number of exports (m) affects the total channel cost. Decisions pertaining to it include the balanced trade-off between exporting cost, holding costs of both the export, and the importer. Figure 10 presents the effect of the number of exports on various cost components when the number of inland shipments (n) is constant at 30.

It can be observed that the exporting cost experiences slight deviation initially due to discounts. However, this effect is lost for higher number of exports. It follows the same drift when the exporting cost increases linearly with the number of exports (m). The holding cost of the

Table 4 Best 25 shipment policies

Solution rank	Numbers of inland shipments (n)	Numbers of exports (m)	Export shipment sizes $r_1, r_2, r_3, \ldots, r_m$	Holding cost of the manufacturing units t	Holding cost of the exporter	Holding cost of the assembly unit	Setup costs	Ordering costs of the exporter	Ordering costs of the assembly plant	Exporting costs	Total channel costs
1	8	7	19, 38, 35, 42, 74, 48, 44	5,195	1,057	529	500	400	560	3,100	11,341
2	8	6	15, 41, 81, 48, 35, 80	5,195	1,275	517	500	400	480	3,000	11,367
3	8	8	11, 42, 38, 47, 38, 45, 37, 42	5,195	785	900	500	400	640	3,200	11,620
4	9	8	17, 29, 40, 43, 29, 32, 70, 40	5,437	987	125	500	450	640	3,500	11,639
5	8	5	29, 63, 46, 84, 78	5,195	1,454	920	500	400	400	2,900	11,769
6	9	7	8, 38, 75, 31, 48, 32, 68	5,437	1,057	429	500	450	560	3,400	11,833
7	14	8	3, 49, 44, 44, 31, 40, 45, 44	5,381	781	663	500	700	640	3,200	11,865
8	12	9	8, 45, 36, 22, 37, 46, 28, 37, 41	5,171	682	678	500	600	720	3,600	11,951
9	12	8	14, 45, 38, 47, 37, 40, 35, 44	5,171	650	1,200	500	600	640	3,200	11,961
10	10	9	5, 38, 42, 35, 31, 22, 44, 47, 36	5,485	768	389	500	500	720	3,600	11,962
11	10	8	19, 18, 34, 75, 31, 33, 47, 43	5,485	856	513	500	500	640	3,500	11,994
12	10	7	9, 60, 47, 38, 48, 49, 49	5,485	862	1,029	500	500	560	3,100	12,036
13	12	7	12, 49, 55, 49, 38, 43, 54	5,171	854	1,057	500	600	560	3,400	12,142
14	9	9	4, 44, 33, 41, 35, 33, 37, 36, 37	5,437	670	767	500	450	720	3,600	12,144
15	14	9	15, 32, 28, 41, 24, 49, 41, 32, 38	5,381	685	567	500	700	720	3,600	12,153
16	12	10	10, 22, 38, 39, 32, 25, 39, 22, 30, 43	5,171	644	470	500	600	800	4,000	12,185
17	14	7	30, 40, 48, 44, 47, 45, 46	5,381	652	1,629	500	700	560	2,800	12,222
18	12	5	41, 49, 84, 49, 77	5,171	1,127	1,860	500	600	400	2,600	12,258
19	9	6	22, 72, 22, 74, 37, 73	5,437	1,065	1,117	500	450	480	3,300	12,349
20	9	5	11, 78, 69, 69, 73	5,437	1,400	1,000	500	450	400	3,200	12,387
21	12	11	12, 19, 45, 13, 22, 46, 12, 25, 39, 37, 30	5,171	635	227	500	600	880	4,400	12,413
22	14	6	25, 46, 75, 45, 49, 60	5,381	900	1,517	500	700	480	3,000	12,478
23	15	8	33, 21, 37, 41, 42, 46, 38, 42	5,604	733	1,075	500	750	640	3,200	12,502
24	14	10	2, 37, 39, 32, 34, 18, 35, 35, 32, 36	5,381	581	580	500	700	800	4,000	12,542
25	10	6	43, 26, 71, 31, 65, 64	5,485	1,000	1,283	500	500	480	3,300	12,548

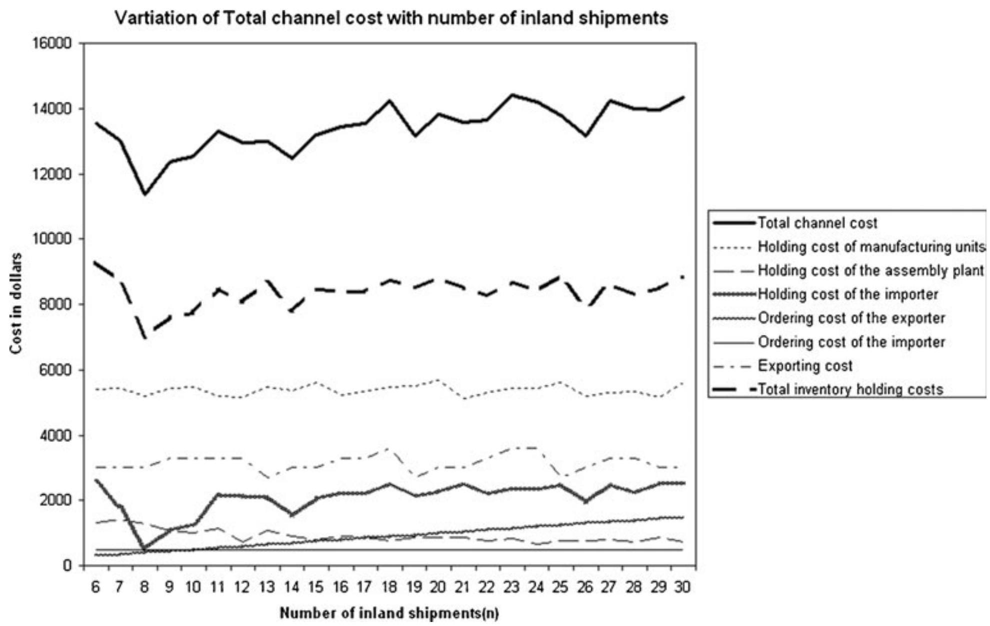


Fig. 9 Variation of costs with number of inland shipments

manufacturing units is only dependent on the inland shipment policy and is unaffected by the number of exports (m). The inventory holding cost of the exporter tends to decrease with the increase in the numbers of exports. It implies that if the exporter dispatches the stock more frequently, reduction in his inventory holding cost is more. The holding cost at the assembly plant is also found to decrease with the increase in the number of exports. When the number of exports is less, more stock is received at the assembly unit increasing its holding cost over time. The ordering cost of the assembly plant is found to increase linearly with number of exports while, the ordering cost of the exporter remained unaffected to the number of exports.

6.3 Effect of export shipment sizes (r_1, r_2, r_3, …, r_m) on the total channel cost

The combination of the export shipment sizes r_1, r_2, r_3, …, r_m plays an important role in reducing the total channel cost. For a given number of inland shipments and exports, there exists a best export shipment policy which determines the each shipment size. This policy is discovered by using the

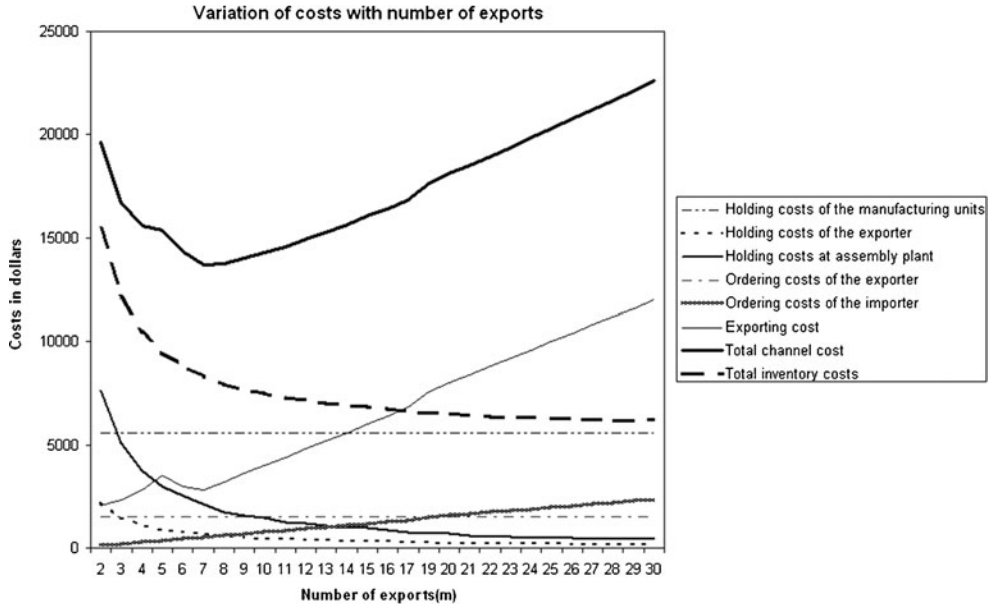


Fig. 10 Variation of costs with number of exports

EPSO technique. EPSO convergence curve of export polices is presented for $n=29$ and $m=18$, (Fig. 11) to illustrate the role of export shipment policy in cutting down the total channel cost.

During the search for best policy, the total channel cost comes down when a better policy or shipment size is found than the existing one. It can be observed in Fig. 11 that the initial policy is found to have the total channel cost of \$17,024, whereas for the same number of inland shipments and exports, the best policy is found to have a cost of \$16,914 after 202 iterations. Hence, the best export policy could reduce channel costs by \$110 (\$17,024–16,914) every month.

7 Conclusions

Three-level inventory outsourcing problem is solved here to support the decision making for the outsourcing firms in the context of logistic operations. The key ingredients of the logistical operations were identified to be the shipping frequency and the shipment sizes. The decisions over these factors derive the trade-off between the inventory holding costs, ordering costs, the transportation costs, and discounts. Two shipping policies namely the inland shipping policy and the export shipping policy have been proposed. The proposed solution methodology successfully finds the optimal combination of the shipping frequencies and shipment sizes for both of the policies. Besides, the results reveal the need to avoid shipment decisions biased by the transportation discount options. The proposed model can effectively be utilized as a tool by the outsourcing firms to handle fluctuations in the possible shipment options. Although the model is presented as a solution for the procurement of outsourced manufacturing components, it can simultaneously be used for the inventory problem of a retailer of imported goods. This could be achieved by a simple assumption that the consumption of components at the assembly plant is analogous to the demand of the retailer/importer.

The proposed outsourcing model is underpinned by some limitations in an attempt to avoid more mathematical complications and restore the central theme of the model. However, good challenges posed by these limitations for future research include the following:

- The procurement cost may vary if the components arrive at different frequencies to the exporter. This limitation suggests further research work in deriving the different optimal delivery frequencies for each of the manufacturing units and their shipment sizes.
- Another good study would be the reformulation of the proposed outsourcing model in order to introduce uncertainty in the shipment frequencies and relaxing the assumptions of deterministic and constant consumption rate of components at the assembly plant.
- This proposed model can be made more general by releasing some of the assumption (Section 2.3) and can be studied for a more complex system.

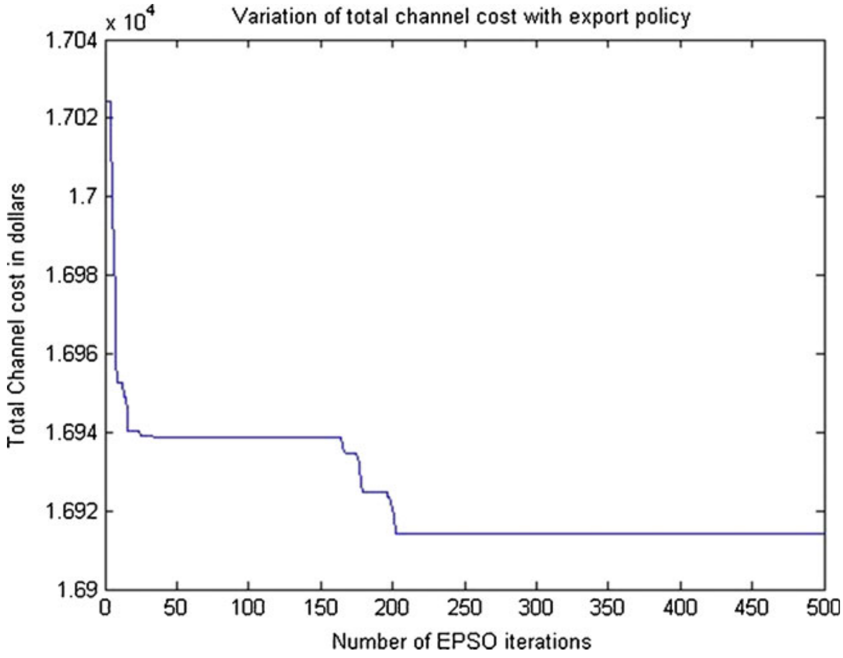


Fig. 11 Convergence of export policies using EPSO

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